QUASILINEAR RELAXATION OF A BEAM IN A PLASMA WITH DECREASING DENSITY

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It is well known [1-3] that the nonuniformity of the density of a plasma strongly affects the character of the interaction of fast-electron beams with it. Until recently, because of the problem of heating a laboratory plasma, primarily flows moving in the direction of increasing plasma density were studied. Here the relaxation of the beam is accompanied by the appearance of accelerated electrons [3].

The case of a beam moving in the direction of decreasing density of the surrounding plasma is just as interesting. A typical example of this physical situation is the motion of electrons accelerated in the region of a solar flare into the outer layers of the coronal plasma. Such an electron flow generates Langmuir waves in the plasma (plasmons), which partially transform into electromagnetic waves and are observed in the sporadic radiation from the sun in the form of bursts of type III [4]. The qualitative features of the beam–plasma interaction with the beam moving in the direction of decreasing plasma density were pointed out in [3-6]. In this case, unlike a beam in a uniform plasma or in a plasma whose density grows along the beam, the phase velocity of plasmons resonant with the beam decreases as the beam propagates into the less dense plasma. In the process, they fall out of resonance with the beam, which retards the process of beam relaxation – the transfer of beam energy to plasma waves.

In this paper we analyze the spatial evolution of the distribution function of a stationary electron beam and the spectrum of plasma waves in a plasma whose density decreases along the beam based on one-dimensional equations describing the spatial evolution of the limits of the quasilinear plateau [1, 7]. It is shown that if in such a plasma the modulus of the density gradient also decreases in the direction of motion of the beam, then the unstable section of the distribution function, responsible for the generation of plasmons, breaks down at significantly larger distances than in a uniform plasma. The beam relaxation length, understood in this sense, is comparable to the characteristic size of the nonuniformity [1, 8, 9].

In addition, the plasmon spectrum changes significantly. The spectral density of the plasmons does not decrease, as it does in a uniform plasma, but rather increases as the wavelength decreases. For this reason Landau damping, as a result of which a maximum forms in the plasmon spectrum in the region of small wavelengths, must be taken into account.

1. Evolution of the Distribution Function of the Beam. For stationary injection of a beam into a nonuniform plasma the spatial evolution of the beam and the plasma waves excited by it is described by the system of quasilinear equations [1-3]:

$$v\frac{\partial f}{\partial x}=\frac{8\pi^2e^2}{m^2}\frac{\partial}{\partial v}\left(\frac{1}{v}W_k\frac{\partial f}{\partial v}\right); \tag{1.1}$$

$$\frac{3v_T^2}{v}\frac{\partial W_k}{\partial x}+\frac{v^2}{2}\frac{\partial}{\partial x}\left(\frac{\omega_p^2}{\omega^2}\right)\frac{\partial W_k}{\partial v}=\frac{\pi\omega_p^2}{\omega n(x)}W_kv^2\left(\frac{\partial f}{\partial v}+\frac{\partial F}{\partial v}\right). \tag{1.2}$$

Here $W_k(x, v)$ is the spectral energy density of the plasmons; $f(x, v)$ is the electron distribution function of the beam; $\omega^2 = 4\pi n(x)e^2/m$; $n(x)$ is the plasma density; $\omega^2 = \omega_p{}^2 + 3k^2v_T{}^2$; k is the wave vector of a plasmon in resonance with the beam electrons; $v = \omega/k$; $F = (n/\sqrt{\pi}v_T)\exp(-v^2/2v_T{}^2)$ is the electron distribution function for the main plasma with temperature T; and, $v_T = \sqrt{\varkappa T/m}$ is the thermal velocity. The starting beam distribution function $f(0, v) = f_0(v)$ and the variation of the plasma density $n(x) = n_0\nu(x)$ are assumed to be given.

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It is assumed that the velocity spread Δv in the beam is large enough for a kinetic instability to develop:

$$(n'/n)^{1/3} \ll \Delta v/v_0, \ (\Delta v/v_0)^2 \ll 1$$

(n' is the beam density and v_0 is the average beam velocity).

As a result of the quasilinear interaction of a one-dimensional beam with a uniform plasma, as is well known [1, 4], a plateau $f = f_n(x)$ forms in the distribution function $f(x, v)$ (the section with $\partial f/\partial v = 0$ for $v_1(x) < v < v_2(x)$). The height of the plateau in the stationary case is determined by the law of conservation of the particle flux:

$$f_n(x) = \frac{2}{v_2^2(x) - v_1^2(x)} \int_{v_1(x)}^{v_2(x)} u f_0(u)\, du. \tag{1.3}$$

This enables describing the evolution of the distribution function based on the equations for the evolution of the boundaries of the plateau $v_1(x)$ and $v_2(x)$. It is obvious that a spectrum of plasma waves $W_k(x, v)$ is excited in the same interval of phase velocities, while outside it $W_k = W_{kT}$ (the spectrum of thermal noise in the plasma) and $f = f_0(v)$.

In a nonuniform plasma, as shown qualitatively in [1], beam relaxation is possible with a relatively weak nonuniformity. An analogous inequality, derived in the Appendix based on an analysis of the relaxation process (A.11), can be written in the form

$$\left|\frac{1}{\nu}\frac{d\nu}{dx}\right| \leqslant (\gamma_0 \Delta v/\Lambda v_0)(v_T/v_0)^2, \tag{1.4}$$

where $\gamma_0 = n'\omega_p v_0^3/[n_0(\Delta v)^2 v_T^2]$ is the spatial increment of the beam instability in a uniform plasma; Λ is the coulomb logarithm [1]. Part of the distribution function of the beam in the process of relaxation has, as in a uniform plasma [10], the form of a plateau.

Near the boundaries of the plateau the distribution function is described by the relations [see (A.4)] [11]

$$f - f(v_{1,2}) = \frac{3v_T^2 n\omega}{\pi\omega_p^2}\left(\frac{1}{6v_T^2}\frac{d\nu}{dx} - \frac{1}{v_{1,2}^3}\frac{dv_{1,2}}{dx}\right)\ln\frac{W_k}{W_{kT}}. \tag{1.5}$$

To obtain the equations of evolution of the plateau boundaries the jumps in the distribution function at the limits $f_n - f_0(v_{1,2})$ must be substituted into the left side of (1.5). Assuming $\ln(W_k/W_{kT}) = \Lambda \approx$ const, we have

$$(3v_T^2/v_1^3)\, dv_1/dx = -(4\pi^2 e^2/m\omega\Lambda)[f_n(x) - f_0(v_1)] - (1/2)|d\nu/dx|; \tag{1.6}$$

$$(3v_T^2/v_2^3)\, dv_2/dx = (4\pi^2 e^2/m\omega\Lambda)[f_0(v_2) - f_n(x)] - (1/2)|d\nu/dx|. \tag{1.7}$$

Near the lower limit $v \sim v_1$ it can be assumed that $f_n \sim n'/v_0 \gg f_0(v_1)$ [1], and the solution of (1.6) can be written as

$$v_1 = v_0/\left[1 + (v_0^2/3v_T^2)(n'\omega x/(n(x)\Delta v\Lambda) + 1 - \nu(x)\right]^{1/2}. \tag{1.8}$$

It is obvious that for sufficiently large distances $x \gtrsim L_0$ ($L_0 = n_0 \Delta v \Lambda v_T^2/(n'\omega v_0^2)$ is the relaxation length in a uniform plasma) $v_1 \sim 1/\sqrt{x}$, as in a uniform plasma [1, 8].

The evolution of the upper limit $v_2(x)$ depends strongly on the gradient of the plasma density. The first term (1.7) is responsible for the generation of plasma waves at the limit of the plateau and expansion of the plateau owing to diffusion, while the second term describes the process of these waves falling out of resonance with the beam, owing to which dv_2/dx also decreases.

Figure 1 shows qualitatively the evolution of the distribution function: slowing down of the relaxation of the "peak" in the starting distribution function f_0 and formation of the plateau f_n with the limits v_1 and v_2 ($v_1{}^0$ and $v_2{}^0$ are the boundaries of the plateau in a uniform plasma).

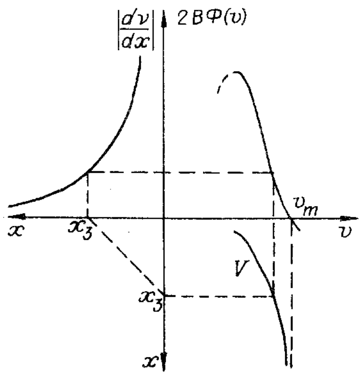

Fig. 1

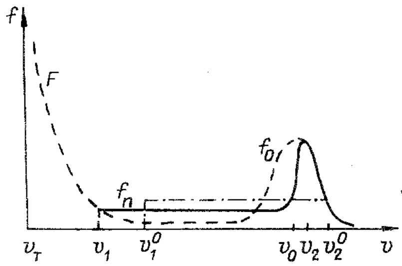

Fig. 2

At large distances ($x \gtrsim L_0$) the analysis can be confined to the evolution of the top limit, since at the bottom limit the velocities are so low that $v_1^2 \ll v_2^2 \sim v_0^2$. As a result of this inequality Eq. (1.7) for the top limit at large distances has the form

$$(3v_T^2/v_2^3)\, dv_2/dx = B\Phi(v_2) - (1/2)\,|d\nu/dx|, \qquad (1.9)$$

where

$$B = \frac{4\pi^2 e^2}{\Lambda m\omega}, \quad \Phi(v) = f_0(v) - \frac{2}{v^2}\int_0^v u f_0(u)\, du. \qquad (1.10)$$

The solution of Eq. (1.9) describes the transfer of electron energy to plasmons only in the case of expansion of the plateau towards high velocities, when $dv_2/dx > 0$, i.e., when the condition $|d\nu/dx| \le 2B\Phi(v_2)$ holds, which is qualitatively analogous to (1.4). We shall seek the solution of (1.9) near the root of its right side:

$$v_2(x) = V(x) + s(x), \quad |s/V| \ll 1, \qquad (1.11)$$

where V(X) is determined by the equation

$$|d\nu/dx| - 2B\Phi(V(x)) = 0. \qquad (1.12)$$

We shall first study the case of a constant density gradient $d\nu/dx = -\varepsilon$, when (1.12) has the root $v_0 \lesssim V = V_c(\varepsilon) < v_m$, whose position does not depend on x. Then the solution of (1.9) can be written in the form of an inverse function:

$$x = 6v_T^2 \int_{v_0}^{v_2(x)} u^{-3} du/[2B\Phi(u) - \varepsilon].$$

Using the expansion (1.11) with $V(x) = V_c$, from (1.9) we find $v_2(x) \cong V_c - s_0 \exp(-x/L_1)$. Here $s_0 = V_c - v_0$, and the characteristic scale L_1 of the evolution of $v_2(x)$ is the same as in a uniform plasma: $L_1 = (3v_T^2/V_c^3)/|Bd\Phi/dv|_{v=V_c} \sim L_0$.

Thus as the beam propagates into a plasma with a linearly decreasing density only some of the beam electrons in the velocity interval $v_0 - \Delta v \lesssim v \lesssim V_c$ give up energy to plasmon generation. The remaining electrons with $v > V_c$ do not interact with the plasma even at larger distances ($x \gg L_0$). Such "partial" relaxation can be regarded as the first small-scale stage of quasilinear interaction of the beam with a nonuniform plasma, when the gradient of the nonuniformity is assumed to be constant. In the general case ($d\nu/dx \ne$ const) the evolution depends on the character of the change in the gradient of the plasma density.

Figure 2 shows schematically the graphical solution of Eq. (1.12), i.e., the determination of $V_3 = V(x_3)$ for the characteristic argument x_3.

For s(x) from (1.9) and (1.11) we obtain with accuracy up to $|s/V| \ll 1$

$$\frac{ds}{dx} - sB\frac{V^3}{3v_T^2}\left(\frac{d\Phi}{dv}\right)_{v=V} = \frac{1}{2}\,\frac{d^2\nu/dx^2}{(Bd\Phi/dv)_{v=V}}. \qquad (1.13)$$

The solution of (1.13) together with (1.12) describes the evolution of $v_2(x)$:

$$v_2(x) = V(x) + \int_{\infty}^{x} \frac{(d^2\nu/dz^2)\,dz}{2B\,(d\Phi/dv)_{v=V(z)}} \exp\left[B\int_{z}^{x} dy\, \frac{V^3(y)}{3v_T^2}\left(\frac{d\Phi}{dv}\right)_{v=V(y)}\right]. \quad (1.14)$$

Since the gradient of the plasma density decreases in a power-law fashion [12], it follows from (1.11) and (1.14) that for $x \gg L_0$

$$v_2(x) = V(x) - \frac{3v_T^2 d^2\nu/dx^2}{2B^2V^3(x)\,(d\Phi/dv)^2_{v=V(x)}}. \quad (1.15)$$

Analogously, the dependence $V(x)$ can also be determined at quite large distances ($x \gg L_0$), when the top limit of the plateau $v_2(x)$ approaches v_m – a root of the function $\Phi(v)$. It can be shown that for continuous nonnegative distribution functions $f_0(v)$, which decrease as $v \to \infty$ more rapidly than v^{-2} (in particular, for a Gaussian distribution or a power-law distribution $v^{-\lambda}$ with $\lambda > 2$), the function $\Phi(v)$ in (1.10) has a unique root $\Phi(v_m) = 0$, and in addition $(d\Phi/dv)_{v=v_m} = (df/dv)_{v=v_m}$. From (1.12) and (1.15) we now find

$$V(x) \cong v_m - |d\nu/dx|/(2Bdf_0/dv)_{v=v_m}; \quad (1.16)$$

$$v_2(x) \approx v_m - |d\nu/dx|/(2B|df_0/dv|_{v=v_m}) - \frac{3v_T^2 d^2\nu/dx^2}{4v_m^3B^2\,(df_0/dv)^2_{v=v_m}}. \quad (1.17)$$

The evolution of $v_2(x)$ determines the relaxation length in a nonuniform plasma: $|(v_2 - v_m)/(dv_2/dx)| \sim |(d\nu/dx)/(d^2\nu/dx^2)| \sim L_n$.

Thus unlike a plasma with a constant density gradient in the case under study the "unperturbed part" of the distribution function of the electrons in the beam for $v > v_2(x)$ decreases on scales of the order of the length of the nonuniformity $L_n \gg L_0$. Generation of plasma waves continues on the same scales. Thus the large-scale evolution of the distribution function of the beam is determined completely by the character of the nonuniformity. For example, in the plasma of the solar corona a slow change in the gradient of the density at characteristic distances of the order of several solar radii R increases the relaxation length of the beam up to a scale of the order of the length of the nonuniformity $L_n \sim R$ [8].

2. Spectrum of Plasma Waves. The behavior of the distribution function and the limits of the plateau in a nonuniform plasma studied above permits studying the spatial evolution of the spectral energy density of the plasma waves W_k [8, 11].

We shall study the dependence $W_k(x, v)$ in the region of high phase velocities $v_T^2 \ll v^2$, when Landau damping is negligibly small. As the plateau is approached we find from (1.1) and (1.2) in the velocity interval $v_1 < v < v_2$ the following linear differential equation for $W_k(x, v)$:

$$\frac{3v_T^2}{v}\frac{\partial W_k}{\partial x} - \frac{v^2}{2}\left|\frac{d\nu}{dx}\right|\frac{\partial W_k}{\partial v} = \frac{mv^3}{2\omega}\frac{df_n}{dx}\left(v^2 - v_1^2(x)\right). \quad (2.1)$$

The method of characteristics permits writing the solution of (2.1) in the form of an integral

$$W_k(x, v) = \frac{m}{6v_T^2\omega}\int_{x_0(V)}^{x} dz\, v^6(z, V)\,\frac{df_n}{dz}\left[1 - \frac{v_1^2(z)}{v^2(z, V)}\right] + W_{kT}, \quad (2.2)$$

where the characteristic is given by the expression $v(x, V) = V/\sqrt{1 + (V^2/3v_T^2)(1 - \nu(x))}$. For $W_k \gg W_{kT}$ the additive constant can be neglected, and the lower limit $x_0(V)$ is the solution of the equation $v(x_0, V) = v_2(x_0)$.

The analytical dependence $W_k(x)$ can also be obtained only at sufficiently large distances (at the final stage of relaxation), when $v_1^2 \ll v_2^2$, while the top limit is close to the limiting value $v_2 \approx v_m$. Under these conditions from (1.3) and (1.16) we obtain

$$\frac{df_n}{dx} \approx \frac{2}{v_2}\Phi(v_2)\frac{dv_2}{dx} = \frac{|d\nu/dx|\, d^2\nu/dx^2}{2B^2 v_m\, |df_0/dv|_{v=v_m}}. \qquad (2.3)$$

Substituting (2.3) into (2.2) and assuming that $d^2\nu/dz^2$ is nearly constant in the interval $x_0 < z < x$, we obtain

$$W_k(x, v) \approx \frac{\Lambda^2 m n_0^2 v_m}{\pi^2 \omega_0\, |df_0/dv|_{v=v_m}} \left(\frac{v_m}{v_0}\right)^2 \frac{d^2\nu}{dx^2}\, \frac{v_2^4(x) - v^4}{v_m^4}. \qquad (2.4)$$

We note that for a power-law model $\nu(x)|_{x \gg L_0} \sim x^{-\alpha} (\alpha > 0)$ the same dependence $W_k(x, v)$ is also obtained to within a factor of the order of unity. As one can see from (2.4), in the presence of a nonuniformity the spectral energy density of the plasmons does not decrease, as in a uniform plasma, but rather, because plasmons are carried off into the region of low velocities, it increases as the phase velocity decreases (Fig. 3). In the region of low phase velocities $v^2 \gtrsim v_T^2$ the term on the right side of (2.1) responsible for Landau damping must be included. The spectral density in this limiting case can be represented with accuracy up to $r_D/L_0 \ll 1$ $(r_D = v_T/\omega_0)$ in the form

$$W_k(x, v) = \frac{\Lambda^2 m n_0^2 v_T^5\, |d\nu/dx|}{v_m \omega_0^4\, |df_0/dv|_{v=v_m}}\, \frac{d^2\nu}{dx^2}\, \frac{v^2}{v_T^2} \exp\left(\frac{v^2}{2v_T^2}\right). \qquad (2.5)$$

As follows from (2.4) and (2.5), because of damping the spectrum of plasma waves in a nonuniform plasma has a maximum at the phase velocity $v_{ext} \approx \sqrt{2}\, v_T \ln^{1/2}\left[\left(\frac{v_m}{v_T}\right)^4 \frac{\omega_p}{v_T\, |d\nu/dx|}\right] \sim v_T$ unlike the spectrum in a uniform plasma, where $v_{ext} \sim v_0$. In addition, the maximum value $W_k(x, v_{ext})$ is a factor of $(v_0/v_T)^2$ smaller than when the nonuniformity is neglected, and decreases with distance in proportion to the decrease in the modulus of the density gradient $|d\nu/dx|$.

We shall now consider the limits of applicability of the solutions (2.4) and (2.5) for the spectrum W_k. The spectrum is valid when the nonuniformity (1.4) is comparatively weak. In the Appendix this inequality is derived as a condition of applicability of the plateau approximation. In addition, the plasma nonuniformity cannot be close to linear, as the restriction (A.13) on $d^2\nu/dx^2$ shows. The solution found cannot be regarded as a correction to the solution in a uniform plasma. For a power-law dependence for $\nu(x)$ we obtain from (A.13)

$$\frac{1}{\nu}\left|\frac{d\nu}{dx}\right| \gg \frac{n'}{n_0}\frac{\omega}{\Delta v}\sqrt{W_{kT}\omega/\Lambda m v_0^3 n'}. \qquad (2.6)$$

In the plasma of the solar corona [12] with $\omega_0 \approx 10^9$ sec^{-1}, $v_T \sim 10^6$ m/sec, for a beam of electrons with $v_0 \approx 10^8$ m/sec, $\Delta v \approx 10^5$ m/sec it follows from (1.4) and (2.6) that

$$10^{-1}(n'/n_0)\,\mathrm{m}^{-1} \ll |d\nu/dx| \ll 10(n'/n_0)\,\mathrm{m}^{-1}. \qquad (2.7)$$

Thus the gradient of the nonuniformity in the lower corona $|d\nu/dx| \sim 10^{-8}$ m^{-1} (it is an order of magnitude higher during flares) satisfies the criterion (2.7) for beams with $10^{-9} < n'/n_0 < 10^{-7}$, so that the nonuniformity plays a significant role in the formation of the plasmon spectrum.

Appendix. The effect of plasmons falling out of resonance with the beam in a nonuniform plasma is most strongly manifested in velocity intervals where the slope of the distribution function $\partial f/dv$ and the gain of plasma waves are maximum. In this connection, in order to evaluate the plasma nonuniformity for which the plateau approximation can be employed, we shall study the system of quasilinear equations (1.1) near the plateau limits, where the distribution function varies most rapidly.

We write (1.1) and (1.2) in terms of dimensionless variables

$$\frac{\partial G}{\partial \xi} = \frac{1}{u}\frac{\partial}{\partial u}\left(\frac{w}{u}\frac{\partial G}{\partial u}\right), \quad \frac{\partial w}{\partial \xi} + u^3\mu\frac{\partial w}{\partial u} = u^3 w\frac{\partial G}{\partial u}. \qquad (A.1)$$

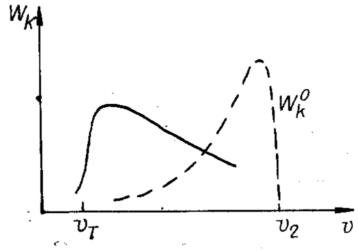

Fig. 3

Here $G = \pi v_0 f/n'$; $\xi = xv_0{}^2 n'\omega/(3v_T{}^2 n_0 v_0)$; $u = v/v_0$; $w = 6\pi\omega v_T{}^2 W_k/(mv_0{}^5 n')$; $\mu = (v_0^2/6v_T^2)\frac{d}{d\xi}\left(\frac{\omega_p^2}{\omega^2}\right)$.

Near the plateau limits we transform into a coordinate system moving together with the limit $\eta = u - u_{1,2}(u_{1,2} = v_{1,2}/v_0)$. Near the limits $|\eta| \ll u_{1,2}$ the form of $G(\xi, \eta)$, $w(\xi, \eta)$ remains virtually unchanged in the process of spatial evolution:

$$\left|\frac{\partial G}{\partial \xi}\right| \ll \left|\frac{du_{1,2}}{d\xi}\frac{\partial G}{\partial \eta}\right|, \quad \left|\frac{\partial w}{\partial \xi}\right| \ll \left|\frac{du_{1,2}}{d\xi}\frac{\partial w}{\partial \eta}\right|. \tag{A.2}$$

Taking into account (A.2), the system (A.1) assumes the form

$$\frac{\partial}{\partial \eta}\left[G + \left(\frac{1}{u_{1,2}^3}\frac{du_{1,2}}{d\xi} - \mu\right)\ln w\right] = 0,$$
$$\frac{\partial}{\partial \eta}\left[\frac{w}{u_{1,2}^2}\frac{\partial G}{\partial \eta} + G\frac{\partial u_{1,2}}{\partial \xi}\right] = 0 \tag{A.3}$$

with the boundary conditions $G(\xi, 0) = G_0(u_{1,2})$, $w(\xi, 0) = w_0$, where $G_0 = \pi v_0 f_0/n'$, $w_0 = 6\pi\omega v_T{}^2 W_{kT}/mv_0{}^5 n'$, f_0 is the unperturbed distribution function of the beam, and W_{kT} is the spectral density of thermal noise in the plasma.

We obtain the solution of Eqs. (A.3) analogously [10] in the implicit form

$$\eta = -\left(w_0/u_{1,2}^2\frac{du_{1,2}}{d\xi}\right)\mathrm{li}\,(w/w_0),\ G(\xi, \eta) - G_0(u_{1,2}) = -\delta(\xi)\ln(w/w_0)$$
$$\left(\mathrm{li}\,(z) = \int_0^z dt/\ln t,\ \delta(\xi) = \frac{1}{u_{1,2}^3}\frac{du_{1,2}}{d\xi} - \mu\right). \tag{A.4}$$

For $\ln(w/w_0) \gg 1$, with logarithmic accuracy we obtain from (A.4)

$$w \approx w_0(\eta/\eta_{1,2})\ln(\eta/\eta_{1,2}),$$
$$G - G_0(u_{1,2}) \approx -\delta(\xi)\ln[(\eta/\eta_{1,2})\ln(\eta/\eta_{1,2})]$$
$$\left(\eta_{1,2} = -w_0\Big/\left(u_{1,2}^2\frac{du_{1,2}}{d\xi}\right)\right). \tag{A.5}$$

The expressions (A.5) are applicable, as one can see, only for $\eta/\eta_{1,2} \gg 1$. In the case $w - w_0 \ll w_0$, when $\eta/\eta_{1,2} \lesssim 1$, the solution (A.4) assumes the form

$$w - w_0 \approx w_0(\exp(\eta/\eta_{1,2}) - 1),\ G - G_0(u_{1,2}) \approx -\delta(\xi)(\exp(\eta/\eta_{1,2}) - 1). \tag{A.6}$$

One can see from (A.5) and (A.6) that the distribution function $G(\xi, \eta)$ varies rapidly in the interval $|\eta| \lesssim |\eta_{1,2}|$, while outside this interval $G(\xi, \eta)$ varies slowly (logarithmically). Thus its behavior near the limits can indeed be approximated by a jump.

Let us examine the applicability of the solutions obtained near the bottom ($\eta = \eta_1 > 0$) and top ($\eta = \eta_2 < 0$) limits separately, since the nonuniformity affects them differently.

It follows from (A.4) that for $\mu < 0$ a necessary condition for the existence of the bottom limit is the inequality $\delta < 0$, i.e.,

$$(1/u_1^3)\,|du_1/d\xi| > |\mu|. \tag{A.7}$$

The function $u_1(\xi)$ was found above [see (1.8)]:

$$u_1^2 = u_0^2/(1 + u_0^2(\xi G_n/\Lambda + 1 - \tilde{\nu}))$$
$$(u_0 = v_{10}/v_0 \sim 1,\ \tilde{\nu} = v_0^2\omega_p^2/6v_T^2\omega^2,\ G_n = \pi v_0 f_n/n'). \tag{A.8}$$

Substituting (A.8) into (A.7) we find in dimensional variables

$$|d\nu/dx| < (\gamma_0 \Delta v/v_0 \Lambda)(v_T/v_0)^2. \tag{A.9}$$

The conditions of applicability of Eqs. (A.3) follow from (A.2) in the form $|\eta_{1,2}| \ll u_{1,2}$, whence for the lower limit we have

$$w_0 \ll u_1^3\,|du_1/d\xi|. \tag{A.10}$$

Using (A.8)-(A.10) we obtain the inequality

$$\frac{v_0}{\omega L_0}\,\frac{1 + (L_0/\beta_0)\,|d\nu/dx|}{1 + x/L_0 + (1 - \nu(x))/\beta_0} \gg W_{kT}\,\omega/mn'v_0^3,\ \beta_0 = 3v_T^2/v_0^2, \tag{A.11}$$

from which for small $x \lesssim L_0 \sim n_0 v_T^2 \Delta v/n' v_0^2 \omega$ there follows the following restriction on the parameters of the beam and plasma:

$$v_0/\omega L_0 \gg \omega W_{kT}/mn'v_0^3 \tag{A.12}$$

[for the characteristic parameters of the plasma of the solar corona (A.12) holds]. At large distances ($x \gg L_0$) the lower limit occurs at such low velocities $u_1 \sim v_T/v_0$ that it stops evolving, since the plasmons are absorbed in the main plasma owing to Landau damping.

Analogously, we write the condition of applicability of the solutions obtained near the top limit in the form $w_0 \ll u_2^3|du_2/d\xi|$. Using the expression for $u_2 = v_2(x)/v_0$ (1.17) we find the restriction on the gradient of the nonuniformity:

$$(1/\nu)\,d^2\nu/dx^2 \gg (4\pi\omega W_{kT}/\Lambda mn'v_0^2)\,(n_0\omega_p/n'\Delta v)^2. \tag{A.13}$$

When (A.13) holds the slope of the top boundary of the plateau is much greater than its slope in the region $v_1 < v < v_2$ and the condition $W_k \gg W_{kT}$ holds.

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NUMERICAL CALCULATION OF RELATIVISTIC MULTIPLE-CAVITY SYSTEMS

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In the calculation of various electrophysical devices which use relativistic electron beams, it is necessary to consider the motion of a beam of charged particles in an external electromagnetic field and the self-fields (irrotational and solenoidal) of the beam. A multiple-cavity klystron is an example of such a device, in which the interaction between the relativistic electron beam and the radiation field in the cavities is significant. The numerical treatment of such processes is based on Maxwell's equations.

In the present paper we describe the numerical algorithms and their computer program implementation in the framework of the package of applied programs ÉRANS [1] for the calculation of a relativistic beam of charged particles moving in extended multiple-cavity systems. The problem is split up into the following subproblems: 1) the calculation of the input cavity into which an ungrouped flux of charged particles enters (oscillations in the cavity are excited and maintained by an external source, such as a current loop); 2) the calculation of the flux in the drift tubes; 3) the calculation of the flux in the relay and output cavities; 4) the joining of the solutions of the first three problems.

The problem is assumed to be axisymmetric and is treated in terms of the cylindrical coordinates r, z, θ, where the motion of the beam is mostly along the axis of symmetry z.

Economy of calculation is the basic criterion used in choosing the numerical algorithms. In carrying through the calculations for different parts of the system, the most significant factors affecting the flux of charged particles for that part of the system are taken into account. Inside the cavities the solenoidal fields $\mathbf{E}^S$, $\mathbf{H}$ are taken into account, where the nonzero components of these fields are E_r^S, E_z^S, and H_θ (the so-called E-field). In the drift tubes we take into account the azimuthal component of the self-magnetic field of the particles beam. External electric and magnetic fields act on the beam over the entire system, as does the irrotational field of the beam. The first five harmonics of the vector potential (see below) are used to calculate the solenoidal fields.

The algorithms allow one to follow transient processes in separate parts of the system; however, in the present paper we will be concerned mainly with steady-state, periodic processes. Our approach is illustrated on a problem of practical interest.

We consider separately the algorithms for the solution of the above subproblems. The discussion is ordered in a convenient way for the description of the algorithms. The problem of calculating the flux of charged particles in a resonant cavity reduces to finding the solution of the complete set of Maxwell's equations

$$\operatorname{div} \mathbf{E} = \rho/\varepsilon_0; \quad (1)$$

$$\operatorname{rot} \mathbf{E} = -\mu_0 \partial \mathbf{H}/\partial t; \quad (2)$$

$$\operatorname{div} \mathbf{H} = 0; \quad (3)$$

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